

# A NEW APPROACH TO PERTURBATIVE THEORY IN THE NONPERTURBATIVE QCD VACUUM.

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## Abstract

Using stochastic quantization method [1], we derive gauge-invariant equations, connecting multilocal vacuum correlators of nonperturbative field configurations, immersed into the quantum background. Three alternative methods of stochastic regularization of these equations are suggested, and the corresponding regularized propagators of a background field are obtained in the lowest order of perturbation theory.

## 1 Introduction.

Recently a new approach to investigation of the nonperturbative content of any field theory, based on the equations for vacuum correlators, derived via stochastic quantization method [1], was suggested [2]. It is closely connected to the Method of Vacuum Correlators [3], in which it is postulated, that the whole information about the QCD vacuum structure is maintained in the full set of irreducible vacuum averages (cumulants).

Within this approach, applied to gauge theories, introducing corresponding generating functionals and using cumulant expansion [4,5], one obtains an infinite set of equations. These equations connect correlators, which contain various number of fields and stochastic Gaussian noise in gauge-invariant way. This approach is especially useful in the theories with nontrivial vacuum structure, e.g. in QCD, since the asymptotics of solutions of these equations

at Langevin time  $t$  tending to infinity yield the values of physical correlators without any assumptions about the model of the vacuum. In [2] the minimal closed set of such equations, corresponding to the Gaussian distribution of fields, was obtained for the case of gluodynamics and investigated in the lowest orders of perturbation theory both in standard, non-gauge-invariant, and gauge-invariant ways.

The next related problem, one needs to solve in the framework of this approach, is the problem of separation of perturbative gluonic contributions in every term of cumulant expansion. To this end we split the total gluonic field into a background and a quantum fluctuation:

$$A_\mu^a = B_\mu^a + gQ_\mu^a, \quad (1)$$

where the principle of this division is unimportant [6]. In particular, the background field may be pure classical, and in this case we come to the problem of quantization of classical solutions, but, generally speaking, the fields  $B_\mu^a$  form a quantum ensemble.

Such a separation was used in [6–8] in order to develop perturbative theory in the confining QCD vacuum, which ensures the area law of an averaged Wilson loop with the value of the string tension, known phenomenologically,  $\sigma \simeq 0.2 \text{GeV}^2$  [9]. In [7] this formalism was applied to the case of QCD at finite temperatures.

It was shown in [6], that the confining background kills all the infra-red singularities, the lowest gluon and ghost corrections to the charge renormalization were calculated, and it was found out, that the usual logarithmic growth of  $\alpha_s(R)$  in the empty space at large distances in the one-loop approximation disappears in the presence of a background. Instead of that, one obtains, that  $\alpha_s(R)$  is saturated at the scale of the inverse excitation mass of the transverse string vibration  $m^2 \sim 2\pi\sigma \sim 1 \text{GeV}^2$ .

In [8] unitary background gauges, where ghosts are either absent or non-propagating, were found, which may help to describe hybrid states in terms of physical polarizations of  $Q_\mu^a$  only.

However, in all the papers [6–8] the nonperturbative background fields were considered as given, and this input was parametrized, e.g. by the full set of cumulants. The main goal of this paper is to derive equations, starting from the Lagrangian, from which the correlators of background fields and of the quantum fluctuations may be obtained simultaneously. It seems natural to apply for this purpose stochastic quantization method [1], since the

background field formalism [10,6], developed within this type of quantization, possesses no-ghost property [11] as well as stochastic quantization of gluodynamics in the empty space [12]. The point is that all the individual stochastic diagrams, contributing to some gauge-invariant quantity may remain finite for  $t \rightarrow +\infty$ . The main idea [13] is to transform the gauge field in such a way, that in the Langevin equation for the new field the projector onto transverse degrees of freedom of gluonic field, standing in the action, is replaced by an invertible matrix. This transformation is required to leave unchanged all the gauge-invariant quantities, and, hence, should be a gauge transformation, but depending on  $t$  (because  $t$ -independent gauge transformations leave the form of the Langevin equation invariant).

It is known [12], that it is not necessary to add a gauge-fixing term into the Langevin equation, since the direct iteration in powers of coupling constant without introducing ghost fields leads to the same results as Faddeev–Popov perturbation theory, because the Langevin time takes the role of a gauge parameter. For example, the coefficient at the projector onto longitudinal degrees of freedom of the gauge field in the free propagator is linearly divergent at  $t$  tending to infinity, but, if one fixes  $t$ , calculates gauge-invariant quantities and then goes to the physical limit,  $t \rightarrow +\infty$ , the divergent terms will cancel each other in the same manner as the terms, depending on the gauge parameter in the framework of the usual approach. That is why, in contrast to the Faddeev–Popov method of quantization, which reproduces correctly only small field fluctuations (since for the case of strong fields Gribov ambiguities arise [14]), Langevin equation does not distort nonperturbative effects.

Furthermore, it turns out, that the properly chosen  $t$ -dependent gauge transformation modifies Langevin equation in such a way, that all the linearly divergent terms disappear completely, which is useful for calculations [13], and, in particular, suggests a method of quantization of non-holonomic systems [15]. This so-called stochastic or Zwanziger gauge fixing procedure was applied in [11] to quantization of gluodynamics in a background, and the  $\beta$ -function in the one-loop approximation was computed. In what follows we shall exploit the Zwanziger term, introduced in the paper [11], to fix the gauge of a quantum fluctuation, but, in contrast to [11], we shall not split Langevin equation into two parts in the sense of loop expansion, but use the total one in order to derive equations for correlators both of a background and of quantum fluctuations. This work is performed in section 2.

In section 3 we present three approaches to stochastic regularization of

the obtained equations. The first two covariant derivative approaches use the methods, suggested in the papers [16–19], while the alternative to them, but also Markovian, third approach is a new one. It is then shown, that all the three types of regularization leads to the properly regularized expressions for the propagator of a background field gluon in the lowest order of perturbation theory, when one neglects perturbative gluonic interactions. The main results of the paper and possible future developments are discussed in the Conclusion.

## 2 Equations for correlators in bilocal approximation.

In this section we present a general method of derivation of an infinite system of equations for correlators of background fields  $B_\mu^a$ , quantum fluctuations  $Q_\mu^a$  and stochastic noise fields  $\eta_\mu^a$  and use it to obtain a minimal closed set of such equations, corresponding to the so-called bilocal approximation, which follows from the assumption, that the grand ensemble of fields is Gaussian, so that all the cumulants, higher than quadratic, are put equal to zero. Lattice data suggest that this approximation has good accuracy in the confining regime of an averaged Wilson loop (for a discussion see the last reference in [3]). This hypothesis about the predominance of bilocal correlations in the vacuum leads to the two alternative methods of investigation of higher correlators: the first one is based on the exact equations, where bilocal and higher correlators are considered on the same footing, while the other is the iterative one, where the values of bilocal correlators, obtained from the minimal system of equations, are then used to calculate three-local correlators and so on. Moreover, in what follows we shall neglect all the quantities higher than of the second order in coupling constant, which means, that the equations to be obtained will not reproduce correctly those of Feynman diagrams, which contain three- and four-perturbative-gluonic vertices. In order to extract explicitly the dependence on the coupling constant, we shall deal below with the fields  $a_\mu^a = gA_\mu^a$ ,  $b_\mu^a = gB_\mu^a$  and  $q_\mu^a = gQ_\mu^a$ .

A known important property of the background field method [10,6,8] is that it is possible to introduce the gauge fixing term for  $q_\mu^a$ , which breaks down the gauge invariance of the partition function under quantum gauge transformations (which leave the background unchanged), but preserves gauge invariance under the so-called background gauge transformations

$$b_\mu \rightarrow U^+(b_\mu + i\partial_\mu)U,$$

$$q_\mu \rightarrow U^+ q_\mu U. \quad (2)$$

This background gauge condition leads to the unique choice of Zwanziger term, ensuring locality and renormalizability of the theory [11], so that the Langevin equation takes the form

$$\dot{a}_\mu^a = (D_\lambda^{(a)} \mathcal{F}_{\lambda\mu}^{(a)})^a + g(D_\mu^{(a)} D_\rho^{(b)} q_\rho)^a - g\eta_\mu^a, \quad (3)$$

where

$$\mathcal{F}_{\mu\nu}^a = \partial_\mu a_\nu^a - \partial_\nu a_\mu^a + f^{abc} a_\mu^b a_\nu^c, \quad (D_\lambda^{(a)} \mathcal{F}_{\lambda\mu})^a = \partial_\lambda \mathcal{F}_{\lambda\mu}^a + f^{abc} a_\lambda^b \mathcal{F}_{\lambda\mu}^c,$$

$$\langle \eta_\mu^a(x, t) \eta_\nu^b(x', t') \rangle = 2\delta_{\mu\nu} \delta^{ab} \delta(x - x') \delta(t - t'), \quad (4)$$

and the sign of  $\eta_\mu^a$  is changed.

Due to (2), all the correlators, containing  $q_\mu^a$  will be gauge-invariant, while for the background field one should use Schwinger gauge  $b_\mu^a(x, t)(x - x_0)_\mu = 0$  (where  $x_0$  is an arbitrary point), in which  $b_\mu$  may be explicitly expressed through  $\mathcal{F}_{\mu\nu}^{(b)}$ :

$$b_\mu(x, t) = \int_{x_0}^x dz_\nu \alpha(z, x) \mathcal{F}_{\nu\mu}(z, t),$$

where  $\alpha(z, x) \equiv \frac{(z - x_0)_\nu}{(x - x_0)_\nu}$ , and here and later in all the integrals of the type  $\int_{x_0}^x$  the path of integration is a straight line. However, the final equations will be gauge-invariant in the same way, as it was discussed in [2].

Introducing the generating functional

$$\Phi_\beta = P \exp i \oint_C dx_\mu \left\{ \int_{x_0}^x dz_\nu \alpha(z, x) \mathcal{F}_{\nu\mu}^{(b)}(z, t) + g \left( q_\mu(x, t) + \beta \int_0^t dt' \eta_\mu(x, t') \right) \right\}, \quad (5)$$

where  $C$  is some fixed closed contour and  $\beta$  is a  $c$ -number, one obtains, using Langevin equation (3):

$$tr \frac{\partial}{\partial t} <\Phi_\beta> = itr \oint_C dx_\mu <\Phi_\beta \left( D_\lambda^{(a)} (\mathcal{F}_{\lambda\mu}^{(a)}(x,t) + g\delta_{\lambda\mu} D_\rho^{(b)} q_\rho(x,t)) + \right. \\ \left. + (\beta-1) g \eta_\mu(x,t) \right) > . \quad (6)$$

Applying to (6) the formula [5]

$$<e^A B> = <e^A> \left( <B> + \sum_{n=1}^{\infty} \frac{1}{n!} \ll A^n B \gg \right), \quad (7)$$

where  $A$  and  $B$  are two arbitrary operators, we have in bilocal approximation:

$$\frac{1}{2} tr \frac{\partial}{\partial t} \ll V_\nu(y, x_0, t) V_\mu(u, x_0, t) \gg = tr \ll V_\nu(y, x_0, t) \left( D_\lambda^{(a)} (\mathcal{F}_{\lambda\mu}^{(a)}(u, x_0, t) + \right. \\ \left. + g\delta_{\lambda\mu} D_\rho^{(b)} q_\rho(u, x_0, t)) \right) \gg, \quad (8)$$

where

$$V_\mu(y, x_0, t) = \int_{x_0}^y dz_\nu \alpha(z, x) \mathcal{F}_{\nu\mu}^{(b)}(z, x_0, t) + g \left( q_\mu(y, x_0, t) + \int_0^t dt' \eta_\mu(y, x_0, t, t') \right), \\ \mathcal{F}_{\nu\mu}^{(b)}(z, x_0, t) = \Phi(x_0, z, t) \mathcal{F}_{\nu\mu}^{(b)}(z, t) \Phi(z, x_0, t), \quad q_\mu(y, x_0, t) = \\ = \Phi(x_0, y, t) q_\mu(y, t) \Phi(y, x_0, t), \quad \eta_\mu(y, x_0, t, t') = \Phi(x_0, y, t) \eta_\mu(y, t') \Phi(y, x_0, t),$$

$$\Phi(z, x_0, t) = Pexp \left[ i \int_{x_0}^z dz'_\sigma b_\sigma(z', t) \right].$$

Noticing, that, due to (1),

$$D_\lambda^{(a)} = D_\lambda^{(b)} - ig[q_\lambda, \cdot], \quad \mathcal{F}_{\lambda\mu}^{(a)} = \mathcal{F}_{\lambda\mu}^{(b)} + g(D_\lambda^{(b)} q_\mu - D_\mu^{(b)} q_\lambda - ig[q_\lambda, q_\mu]),$$

one gets from (8) the first equation of bilocal approximation:

$$\begin{aligned} \frac{1}{2} \text{tr} \frac{\partial}{\partial t} < V_\nu(y, x_0, t) V_\mu(u, x_0, t) > = \text{tr} \frac{\partial}{\partial u_\lambda} \left( < V_\nu(y, x_0, t) \mathcal{F}_{\lambda\mu}^{(b)}(u, x_0, t) > + \right. \\ & \left. + g \frac{\partial}{\partial u_\rho} < V_\nu(y, x_0, t) G_{\rho\lambda\mu}(u, x_0, t) > \right), \end{aligned} \quad (9)$$

where

$$G_{\rho\lambda\mu}(u, x_0, t) = \delta_{\rho\lambda} q_\mu(u, x_0, t) - \delta_{\rho\mu} q_\lambda(u, x_0, t) + \delta_{\mu\lambda} q_\rho(u, x_0, t),$$

and we put all the terms with space-time derivatives to the right hand side. Here in order to disentangle the averages, containing covariant derivatives, we used the formulae:

$$\begin{aligned} \text{tr}(D_\mu^{(b)} M(u, x_0) N) = \text{tr} \left\{ \frac{\partial}{\partial u_\mu} M(u, x_0) N + i \int_{x_0}^u dz_\sigma \alpha(z, u) \cdot \right. \\ \left. \cdot \left( M(u, x_0) N \mathcal{F}_{\mu\sigma}^{(b)}(z, x_0, t) - M(u, x_0) \mathcal{F}_{\mu\sigma}^{(b)}(z, x_0, t) N \right) \right\}, \end{aligned} \quad (10)$$

$$\begin{aligned} \text{tr}(D_\mu^{(b)} D_\nu^{(b)} M(u, x_0) N) = \text{tr} \left\{ \frac{\partial^2}{\partial u_\mu \partial u_\nu} M(u, x_0) N + i \left( M(u, x_0) N \mathcal{F}_{\nu\mu}^{(b)}(u, x_0, t) - \right. \right. \\ \left. \left. - M(u, x_0) \mathcal{F}_{\nu\mu}^{(b)}(u, x_0, t) N \right) + i \int_{x_0}^u dz_\sigma \left( \alpha(z, u) \frac{\partial}{\partial u_\nu} \left( M(u, x_0) N \mathcal{F}_{\mu\sigma}^{(b)}(z, x_0, t) - \right. \right. \right. \\ \left. \left. \left. - M(u, x_0) \mathcal{F}_{\mu\sigma}^{(b)}(z, x_0, t) N \right) + \frac{\partial}{\partial u_\mu} \alpha(z, u) \left( M(u, x_0) N \mathcal{F}_{\nu\sigma}^{(b)}(z, x_0, t) - \right. \right. \right. \\ \left. \left. \left. - M(u, x_0) \mathcal{F}_{\nu\sigma}^{(b)}(z, x_0, t) N \right) \right) + \int_{x_0}^u dz_\sigma \alpha(z, u) \int_{x_0}^u dw_\zeta \alpha(w, u) \cdot \right. \\ \left. \cdot \left( M(u, x_0) \mathcal{F}_{\mu\sigma}^{(b)}(z, x_0, t) N \mathcal{F}_{\nu\zeta}^{(b)}(w, x_0, t) + M(u, x_0) \mathcal{F}_{\nu\zeta}^{(b)}(w, x_0, t) N \mathcal{F}_{\mu\sigma}^{(b)}(z, x_0, t) - \right. \right. \\ \left. \left. - M(u, x_0) \mathcal{F}_{\nu\zeta}^{(b)}(w, x_0, t) \mathcal{F}_{\mu\sigma}^{(b)}(z, x_0, t) N - \right. \right. \end{aligned}$$

$$-M(u, x_0)N\mathcal{F}_{\mu\sigma}^{(b)}(z, x_0, t)\mathcal{F}_{\nu\zeta}^{(b)}(w, x_0, t)\Big)\Big\}, \quad (11)$$

where  $M(u, x_0)$  is equal to  $\mathcal{F}_{\alpha\beta}^{(b)}(u, x_0, t)$  or  $q_\alpha(u, x_0, t)$ ,  $N \equiv \equiv N_{\mu_1 \dots \mu_n}(x_1, t_1, \dots, x_n, t_n, x_0, t)$  is, generally speaking, a product of some number of  $F_{\alpha\beta}$ ,  $q_\alpha$  and  $\eta_\alpha$ , which are given in the points  $x_1 \neq u, \dots, x_n \neq u$  at the moments  $t_1, \dots, t_n$  of Langevin time respectively, where all the parallel transporters between  $x_0$  and each of these points are built of the field  $b_\mu^a$  and given at the same moment  $t$ .

Differentiating equation (6) twice by  $\beta$ , putting then  $\beta$  equal to 1, using the formulae (7), (10) and (11) and the definitions of three- and fourlocal path-ordered cumulants [4,2], one obtains two more equations of bilocal approximation, where all the perturbative correlators, higher than of the second order in coupling constant are neglected:

$$\begin{aligned} & \text{tr} \left\{ \frac{\partial}{\partial t} \langle V_\nu(y, x_0, t) \eta_\mu(u, x_0, t, t') \rangle - \right. \\ & - \frac{1}{2} \oint_C dv_\xi \oint_C dw_\sigma \left( \frac{\partial}{\partial t} \langle V_\xi(v, x_0, t) V_\sigma(w, x_0, t) \rangle \right) \langle V_\nu(y, x_0, t) \eta_\mu(u, x_0, t, t') \rangle + \\ & + g \int_{x_0}^u dz_\sigma \alpha(z, u) \int_{x_0}^u dw_\zeta(w, u) \left( \langle G_{\rho\lambda\mu}(u, x_0, t) \mathcal{F}_{\rho\zeta}^{(b)}(w, x_0, t) \rangle \right. \\ & \cdot \langle \mathcal{F}_{\lambda\sigma}^{(b)}(z, x_0, t) \eta_\nu(y, x_0, t, t') \rangle + \\ & + \langle G_{\rho\lambda\mu}(u, x_0, t) \eta_\nu(y, x_0, t, t') \rangle \langle \mathcal{F}_{\lambda\sigma}^{(b)}(z, x_0, t) \mathcal{F}_{\rho\zeta}^{(b)}(w, x_0, t) \rangle - \\ & - \langle G_{\rho\lambda\mu}(u, x_0, t) \mathcal{F}_{\lambda\sigma}^{(b)}(z, x_0, t) \rangle \langle \eta_\nu(y, x_0, t, t') \mathcal{F}_{\rho\zeta}^{(b)}(w, x_0, t) \rangle - \\ & - \langle G_{\rho\lambda\mu}(u, x_0, t) \mathcal{F}_{\rho\zeta}^{(b)}(w, x_0, t) \rangle \langle \eta_\nu(y, x_0, t, t') \mathcal{F}_{\lambda\sigma}^{(b)}(z, x_0, t) \rangle + \\ & + g^2 \int_{x_0}^u dz_\sigma \alpha(z, u) \left( \langle G_{\rho\lambda\mu}(u, x_0, t) \mathcal{F}_{\rho\sigma}^{(b)}(z, x_0, t) \rangle \langle \eta_\nu(y, x_0, t, t') q_\lambda(u, x_0, t) \rangle + \right. \end{aligned}$$

$$\begin{aligned}
& + < G_{\rho\lambda\mu}(u, x_0, t) q_\lambda(u, x_0, t) > < \eta_\nu(y, x_0, t, t') \mathcal{F}_{\rho\sigma}^{(b)}(z, x_0, t) > - \\
& - < G_{\rho\lambda\mu}(u, x_0, t) \eta_\nu(y, x_0, t, t') > < q_\lambda(u, x_0, t) \mathcal{F}_{\rho\sigma}^{(b)}(z, x_0, t) > - \\
& - < G_{\rho\lambda\mu}(u, x_0, t) \mathcal{F}_{\rho\sigma}^{(b)}(z, x_0, t) > < q_\lambda(u, x_0, t) \eta_\nu(y, x_0, t, t') > \Big) \Big\} = \\
& = \text{tr} \left\{ g \frac{\partial^2}{\partial u_\lambda \partial u_\lambda} < q_\mu(u, x_0, t) \eta_\nu(y, x_0, t, t') > + \right. \\
& \left. + \frac{\partial}{\partial u_\lambda} < \mathcal{F}_{\lambda\mu}^{(b)}(u, x_0, t) \eta_\nu(y, x_0, t, t') > \right\}. \quad (12) \\
& \text{tr} \left\{ \int_0^t dt'' \frac{\partial}{\partial t} < \eta_\nu(y, x_0, t, t'') \eta_\mu(u, x_0, t, t') > + < \eta_\nu(y, x_0, t, t) \eta_\mu(u, x_0, t, t') > - \right. \\
& \left. - < \eta_\nu(y, x_0, t, t') \eta_\mu(u, x_0, t, t) > - \right. \\
& \left. - \frac{1}{2} \oint_C dv_\xi \oint_C dw_\sigma \left( \frac{\partial}{\partial t} < V_\xi(v, x_0, t) V_\sigma(w, x_0, t) > \right) \cdot \right. \\
& \left. \cdot \int_0^t dt'' < \eta_\nu(y, x_0, t, t'') \eta_\mu(u, x_0, t, t') > + \oint_C dz_\sigma \int_0^t dt'' \left( \int_{x_0}^u dw_\zeta \alpha(w, u) \cdot \right. \right. \\
& \left. \left. \cdot < \mathcal{F}_{\lambda\mu}^{(b)}(u, x_0, t) \eta_\sigma(z, x_0, t, t'') > < \eta_\nu(y, x_0, t, t') \mathcal{F}_{\lambda\zeta}^{(b)}(w, x_0, t) > - \right. \right. \\
& \left. \left. - < \mathcal{F}_{\lambda\mu}^{(b)}(u, x_0, t) \mathcal{F}_{\lambda\zeta}^{(b)}(w, x_0, t) > < \eta_\sigma(z, x_0, t, t'') \eta_\nu(y, x_0, t, t') > \right) + \right. \\
& \left. + g \left( < G_{\rho\lambda\mu}(u, x_0, t) \eta_\sigma(z, x_0, t, t'') > < \eta_\nu(y, x_0, t, t') \mathcal{F}_{\rho\lambda}^{(b)}(u, x_0, t) > + \right. \right. \\
\end{aligned}$$

$$+2 < q_\lambda(u, x_0, t) \mathcal{F}_{\mu\lambda}^{(b)}(u, x_0, t) > < \eta_\sigma(z, x_0, t, t'') \eta_\nu(y, x_0, t, t') > \Big) \Big) =$$

$$= g \oint_C dz_\sigma \int_0^t dt'' \left( \int_{x_0}^u dw_\zeta \left( \alpha(w, u) \frac{\partial}{\partial u_\rho} \left( < G_{\rho\lambda\mu}(u, x_0, t) \mathcal{F}_{\lambda\zeta}^{(b)}(w, x_0, t) > \cdot \right. \right. \right.$$

$$\cdot < \eta_\sigma(z, x_0, t, t'') \eta_\nu(y, x_0, t, t') > - < G_{\rho\lambda\mu}(u, x_0, t) \eta_\sigma(z, x_0, t, t'') > \cdot$$

$$\cdot < \eta_\nu(y, x_0, t, t') \mathcal{F}_{\lambda\zeta}^{(b)}(w, x_0, t) > \Big) + \frac{\partial}{\partial u_\lambda} \alpha(w, u) \left( < G_{\rho\lambda\mu}(u, x_0, t) \cdot \right.$$

$$\cdot \mathcal{F}_{\rho\zeta}^{(b)}(w, x_0, t) > < \eta_\sigma(z, x_0, t, t'') \eta_\nu(y, x_0, t, t') > -$$

$$- < G_{\rho\lambda\mu}(u, x_0, t) \eta_\sigma(z, x_0, t, t'') > < \eta_\nu(y, x_0, t, t') \mathcal{F}_{\rho\zeta}^{(b)}(w, x_0, t) > \Big) \Big) +$$

$$+ g \left( \frac{\partial}{\partial u_\rho} < G_{\rho\lambda\mu}(u, x_0, t) \eta_\sigma(z, x_0, t, t'') > \right) < \eta_\nu(y, x_0, t, t') q_\lambda(u, x_0, t) > -$$

$$- g < q_\lambda(u, x_0, t) \eta_\sigma(z, x_0, t, t'') > \frac{\partial}{\partial u_\rho} < \eta_\nu(y, x_0, t, t') G_{\rho\lambda\mu}(u, x_0, t) > \Big). \quad (13)$$

As was discussed in [2], in the physical limit,  $t \rightarrow +\infty$ , in the confining regime of an averaged Wilson loop, the dependence on the point  $x_0$  is negligible, since the difference between each of the cumulants and its gauge-invariant analog is of the order of  $\frac{T_g^2}{R^2} \leq 0.04$  [2], where  $T_g$  is the correlation length of the vacuum, at which a cumulant vanishes and  $R$  is a space width of a Wilson loop, and we obtain gauge-invariant equations for correlators of the fields  $\mathcal{F}_{\mu\nu}^{(b)}$  and  $q_\mu, \eta_\mu$ , immersed into a background as insertions in the parallel transporters.

Note, that among these equations only the first one, equation (9), is linear, while equations (12) and (13) produce complicated hierarchy of perturbative

correlators up to the second order of perturbation theory. Hence, the procedure of solution of equations (9),(12) and (13) is the following: first one needs to put all the perturbative fluctuations equal to zero and to solve the so-reduced equations for correlators of a background and of the Gaussian noise fields (which are just the equations (17),(19) and (20) from the paper [2]). After that one should to include perturbative interactions, expanding the correlators, containing  $q_\mu$  up to the order of  $g^2$  and using the obtained values of pure background and noise fields' correlators.

### 3 Stochastic regularization and perturbative expansion of equations (9),(12) and (13).

In this section we present three methods of stochastic regularization of equations (9),(12) and (13) and use them to derive regularized propagators of a background in the lowest order of perturbation theory in the absence of perturbative corrections. The first two of them are based on covariant derivative regularization schemes, which were suggested in the papers [16–18] and used in [19] to calculate  $\beta$ -function in QCD in the one-loop approximation.

First is the so-called power-law regularization scheme

$$\eta_\mu^a(x, t) \rightarrow \int dy R^{ab}(x, y, t) \eta_\mu^b(y, t),$$

where

$$R^{ab}(x, y, t) = \left[ \frac{1}{(1 - \frac{\Delta}{\Lambda^2})^n} \right]^{ab}(x, y, t), \quad n = 1, 2, \dots,$$

$\Lambda$  is an ultraviolet cutoff,  $\Delta^{ab}(x, y, t) \equiv \int dz (D_\mu)^{ac}(x, z, t) (D_\mu)^{cb}(z, y, t)$  is the covariant Laplacian with  $(D_\mu)^{ab}(x, y, t) \equiv D_\mu^{(b)ab}(x, t) \delta(x - y)$ . One of the main results of the paper [17] is the proof, that any Yang–Mills theory in  $d$  dimensions is regularized to all the orders, when we choose  $n \geq [\frac{1}{2}(d + 1)]$ , where  $[x]$  is the largest integer less than or equal to  $x$ .

The generalization of equations (9), (12) and (13) after applying such a regularization is obvious. For example, equation (9), written in details, takes the form:

$$\frac{1}{2} \text{tr} \frac{\partial}{\partial t} \left\{ \int_{x_0}^y dz_\lambda(z, y) \int_{x_0}^u dx_\rho \alpha(x, u) \langle \mathcal{F}_{\lambda\nu}^{(b)}(z, x_0, t) \mathcal{F}_{\rho\mu}^{(b)}(x, x_0, t) \rangle + \right.$$

$$\begin{aligned}
& +g \int_{x_0}^y dz_\lambda \alpha(z, y) \left( \langle \mathcal{F}_{\lambda\nu}^{(b)}(z, x_0, t) q_\mu(u, x_0, t) \rangle + \right. \\
& \left. + \int_0^t dt' \int dw \langle \mathcal{F}_{\lambda\nu}^{(b)}(z, x_0, t) \xi_\mu(u, w, x_0, t, t') \rangle \right) + g \int_{x_0}^u dx_\rho \alpha(x, u) \left( \langle q_\nu(y, x_0, t) \cdot \right. \\
& \cdot \mathcal{F}_{\rho\mu}^{(b)}(x, x_0, t) \rangle + \int_0^t dt' \int dw \langle \xi_\nu(y, w, x_0, t, t') \mathcal{F}_{\rho\mu}^{(b)}(x, x_0, t) \rangle \left. \right) + \\
& + g^2 \langle q_\nu(y, x_0, t) q_\mu(u, x_0, t) \rangle + g^2 \int_0^t dt' \int dw \cdot \\
& \cdot \left( \langle q_\nu(y, x_0, t) \xi_\mu(u, w, x_0, t, t') \rangle + \langle \xi_\nu(y, w, x_0, t, t') q_\mu(u, x_0, t) \rangle + \right. \\
& \left. + \int_0^t dt'' \int dv \langle \xi_\nu(y, w, x_0, t, t') \xi_\mu(u, v, x_0, t, t'') \rangle \right) \Big\} = \\
& = \text{tr} \frac{\partial}{\partial u_\lambda} \left\{ \int_{x_0}^y dz_\sigma \alpha(z, y) \left( \langle \mathcal{F}_{\sigma\nu}^{(b)}(z, x_0, t) \mathcal{F}_{\lambda\mu}^{(b)}(u, x_0, t) \rangle + \right. \right. \\
& + g \frac{\partial}{\partial u_\lambda} \langle \mathcal{F}_{\sigma\nu}^{(b)}(z, x_0, t) q_\mu(u, x_0, t) \rangle - g \frac{\partial}{\partial u_\mu} \langle \mathcal{F}_{\sigma\nu}^{(b)}(z, x_0, t) q_\lambda(u, x_0, t) \rangle + \\
& + g \delta_{\lambda\mu} \frac{\partial}{\partial u_\rho} \langle \mathcal{F}_{\sigma\nu}^{(b)}(z, x_0, t) q_\rho(u, x_0, t) \rangle \Big) + g \langle q_\nu(y, x_0, t) \mathcal{F}_{\lambda\mu}^{(b)}(u, x_0, t) \rangle + \\
& + g \int_0^t dt' \int dw \langle \xi_\nu(y, w, x_0, t, t') \mathcal{F}_{\lambda\mu}^{(b)}(u, x_0, t) \rangle + \\
& + g^2 \left( \frac{\partial}{\partial u_\lambda} \langle q_\nu(y, x_0, t) q_\mu(u, x_0, t) \rangle - \frac{\partial}{\partial u_\mu} \langle q_\nu(y, x_0, t) q_\lambda(u, x_0, t) \rangle + \right. \\
& \left. + \delta_{\lambda\mu} \frac{\partial}{\partial u_\rho} \langle q_\nu(y, x_0, t) q_\rho(u, x_0, t) \rangle \right)
\end{aligned}$$

$$\begin{aligned}
& + \int_0^t dt' \int dw \left( \frac{\partial}{\partial u_\lambda} < \xi_\nu(y, w, x_0, t, t') q_\mu(u, x_0, t) > - \right. \\
& \left. - \frac{\partial}{\partial u_\mu} < \xi_\nu(y, w, x_0, t, t') q_\lambda(u, x_0, t) > + \right. \\
& \left. + \delta_{\lambda\mu} \frac{\partial}{\partial u_\rho} < \xi_\nu(y, w, x_0, t, t') q_\rho(u, x_0, t) > \right) \Big) \Big) ,
\end{aligned}$$

where  $\xi_\mu(u, w, x_0, t, t') \equiv \Phi(x_0, u, t) \left[ \frac{1}{(1 - \frac{\Delta}{\Lambda^2})^n} \right]^{ab} (u, w, t') \eta_\mu^b(w, t') t^a \Phi(w, x_0, t)$ .

The equations (12) and (13) changes correspondingly.

In the lowest order of perturbation theory, when one neglects perturbative fluctuations, and equations (9), (12) and (13) reduce to the equations for background fields only, one may check in the same way, as it was done in [2] for the unregularized case, that for  $n = 2(d = 4)$  the regularized propagator, obtained from these equations, has the form:

$$\begin{aligned}
< B_\mu^a(x, t) B_\nu^b(y, t) > = & \delta^{ab} \int \frac{dk}{(2\pi)^4} e^{-ik(x-y)} \frac{(\Lambda^2)^4}{k^2(k^2 + \Lambda^2)^4} \left( (1 - e^{-2k^2 t}) T_{\mu\nu} + \right. \\
& \left. + 2k^2 t L_{\mu\nu} \right), \tag{14}
\end{aligned}$$

where  $T_{\mu\nu} \equiv \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$ ,  $L_{\mu\nu} \equiv \frac{k_\mu k_\nu}{k^2}$  are the transverse and the longitudinal projectors respectively. In (14) one may recognize the expression (2.19) from the paper [17], where the "gauge-fixing parameter"  $\alpha = 2k^2 t$ , which seems to be natural in the sense, stated in the Introduction.

Second method of regularization exploits the so-called heat-kernel regularization scheme [18], where the regulator has the form  $R^{ab}(x, y, t) = = (e^{\frac{\Delta}{\Lambda^2}})^{ab}(x, y, t)$ , that is believed to be technically superior for nonperturbative analysis. In analogous way, one gets from the equations (9) and (12) in the lowest order, when the perturbative gluons' contributions are neglected:

$$< B_\mu^a(x, t) B_\nu^b(y, t) > = \delta^{ab} \int \frac{dk}{(2\pi)^4} e^{-ik(x-y)} \frac{e^{-\frac{2k^2}{\Lambda^2}}}{k^2} \left( (1 - e^{-2k^2 t}) T_{\mu\nu} + 2k^2 t L_{\mu\nu} \right), \tag{15}$$

which coincides with the formula (9) from the paper [18] at  $\alpha = 2k^2 t$ .

Finally, let us present a new one, also Markovian and preserving gauge invariance, type of regularization <sup>1</sup>. Its basic idea is to smear space-time delta-function in (4) so, that properly modified Langevin equation remains gauge-invariant. To this end we introduce the new noise fields

$$\xi_\mu^a(x, t) \equiv \frac{(\Lambda^2)^2}{4\pi^2} \int dy e^{-\frac{\Lambda^2(x-y)^2}{2}} \eta_\mu^a(y, t), \quad (16)$$

so that

$$\langle \xi_\mu^a(x, t) \xi_\nu^b(y, t') \rangle = \frac{(\Lambda^2)^2}{8\pi^2} \delta_{\mu\nu} \delta^{ab} \delta(t - t') e^{-\frac{\Lambda^2(x-y)^2}{4}}, \quad (17)$$

$$\lim_{|\Lambda| \rightarrow +\infty} \langle \xi_\mu^a(x, t) \xi_\nu^b(y, t') \rangle = 2\delta_{\mu\nu} \delta^{ab} \delta(x - y) \delta(t - t')$$

and modify the Langevin equation (3) in gauge-invariant way

$$\begin{aligned} \dot{a}_\mu^a &= \left( D_\lambda^{(a)} \mathcal{F}_{\lambda\mu}^{(a)} \right)^a + g \left( D_\mu^{(a)} D_\rho^{(b)} q_\rho \right)^a - \\ &- \frac{g(\Lambda^2)^2}{4\pi^2} \int dy e^{-\frac{\Lambda^2(x-y)^2}{2}} \Phi(x, y, t) \eta_\mu(y, t) \Phi(y, x, t), \end{aligned} \quad (18)$$

keeping in mind, that at  $|\Lambda| \rightarrow +\infty$  the integral in the right hand side is saturated at  $|y - x| \ll \frac{1}{|\Lambda|}$ .

Then, due to (16) and (17), in the lowest order of perturbation theory in the absence of perturbative gluons, it follows from the equations (9) and (12), regularized according to (18), correspondingly

$$\frac{1}{2} \frac{\partial}{\partial t} \left( \langle B_\nu^a(y, t) B_\mu^b(u, t) \rangle + \frac{(\Lambda^2)^2}{4\pi^2} \int_0^t dt' \int dv \left( e^{-\frac{\Lambda^2(u-v)^2}{2}} \langle B_\nu^a(y, t) \eta_\mu^b(v, t') \rangle + \right. \right.$$

$$\left. \left. + e^{-\frac{\Lambda^2(y-v)^2}{2}} \langle B_\mu^a(u, t) \eta_\nu^b(v, t') \rangle \right) \right) + \frac{(\Lambda^2)^4}{32\pi^4} \int_0^t dt' \int dv dv' e^{-\frac{\Lambda^2((y-v)^2 + (u-v')^2)}{2}}.$$

$$\cdot \left( \langle \eta_\nu^a(v, t) \eta_\mu^b(v', t') \rangle + \langle \eta_\nu^a(v, t') \eta_\mu^b(v', t) \rangle \right) = \left( \frac{\partial^2}{\partial u_\rho \partial u_\mu} \delta_{\mu\lambda} - \frac{\partial^2}{\partial u_\mu \partial u_\lambda} \right).$$

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<sup>1</sup>This method was suggested by Professor Yu.A.Simonov.

$$\cdot \left( \langle B_\lambda^a(u, t) B_\nu^b(y, t) \rangle + \frac{(\Lambda^2)^2}{4\pi^2} \int_0^t dt' \int dv e^{-\frac{\Lambda^2(y-v)^2}{2}} \langle B_\lambda^a(u, t) \eta_\nu^b(v, t') \rangle \right), \quad (19)$$

$$\int dv e^{-\frac{\Lambda^2(u-v)^2}{2}} \frac{\partial}{\partial t} \langle B_\nu^a(y, t) \eta_\mu^b(v, t') \rangle + \left( \frac{\partial^2}{\partial u_\mu \partial u_\lambda} - \frac{\partial^2}{\partial u_\rho \partial u_\rho} \delta_{\mu\lambda} \right) \cdot \int dv e^{-\frac{\Lambda^2(y-v)^2}{2}} \langle B_\lambda^a(u, t) \eta_\nu^b(v, t') \rangle = -\frac{1}{2} \delta_{\mu\nu} \delta^{ab} \delta(t - t') e^{-\frac{\Lambda^2(y-u)^2}{4}}. \quad (20)$$

Looking for  $\langle B_\nu^a(y, t) \eta_\mu^b(u, t') \rangle$  in the form  $\delta^{ab} d_{\mu\nu}(z, \tau)$ , where  $z = u - y, \tau = |t - t'|, d_{\nu\mu}(z, \tau) = d_{\mu\nu}(z, \tau), d_{\mu\nu}(-z, \tau) = d_{\mu\nu}(z, \tau)$ , one obtains from (20):

$$\bar{d}_{\mu\nu}(k, \tau) = -\frac{8\pi^2}{(\Lambda^2)^2} \theta(\tau) e^{-\frac{k^2}{\Lambda^2}} (T_{\mu\nu} e^{-k^2\tau} + L_{\mu\nu}), \quad (21)$$

where  $\bar{d}_{\mu\nu}(x, \tau) \equiv \int dy e^{-\frac{\Lambda^2(x+y)^2}{2}} d_{\mu\nu}(y, \tau)$ .

Looking for  $\langle B_\nu^a(y, t) B_\mu^b(u, t) \rangle$  in the form  $\delta^{ab} h_{\mu\nu}(z, t)$ , where  $h_{\nu\mu}(z, t) = h_{\mu\nu}(z, t), h_{\mu\nu}(-z, t) = h_{\mu\nu}(z, t)$  and using (4) and (20), one gets from the equation (19):

$$\left( \frac{1}{2} \delta_{\mu\lambda} \frac{\partial}{\partial t} + \frac{\partial}{\partial z_\mu \partial z_\lambda} - \frac{\partial}{\partial z_\rho \partial z_\rho} \delta_{\mu\lambda} \right) h_{\lambda\nu}(z, t) = -\frac{(\Lambda^2)^2}{4\pi^2} \bar{d}_{\mu\nu}(z, 0),$$

and, hence, due to (21), the regularized propagator of a background field has the form

$$\langle B_\mu^a(x, t) B_\nu^b(y, t) \rangle = \delta^{ab} \int \frac{dk}{(2\pi)^4} e^{-ik(x-y)} \frac{e^{-\frac{k^2}{\Lambda^2}}}{k^2} \left( (1 - e^{-2k^2 t}) T_{\mu\nu} + 2k^2 t L_{\mu\nu} \right). \quad (22)$$

Therefore, we see, that this method of regularization is similar to the heat-kernel method, but is simpler, since there do not arise higher derivatives in the regulator.

## 4 Conclusion

In this paper we applied stochastic quantization [1] to develop a method of derivation of an infinite system of exact equations for gauge-invariant correlators in gluodynamics, where all the perturbative contributions are extracted explicitly in the form of insertions into background parallel transporters. Therefore, the obtained equations allow one to derive pure background correlators and the correlators, containing perturbative corrections, simultaneously, using for quantization the same stochastic noise fields.

After that we obtained the minimal set of equations of bilocal approximation (corresponding to the Gaussian distribution of fields), where we threw away all the perturbative interactions higher than of the second order, and suggested for it three methods of stochastic regularization, all of which preserve gauge invariance of the obtained equations. The first two of them, based on the so-called covariant derivative regularization schemes, lead, in the lowest order of perturbative theory in the absence of perturbative gluons, to the known values of the regularized propagator of a background field, while the third method is a new one. It yields the results, similar to the heat-kernel method, but is simpler than the latter, since in the framework of this method higher derivatives in the regulator do not exist.

The application of the suggested approach to treating the large- $N$  regime of QCD as well as the new equation for the Master field and its connection with the Bootstrap equation will be a topic of a separate publication. Possible types of solutions of the derived equations will be presented elsewhere.

The results, presented in this paper, were partially reported at the International Workshop "Nonperturbative Approaches to QCD", Trento, Italy, July 10–29, 1995.

## 5 Acknowledgements

I am grateful to Professor Yu.A.Simonov for useful discussions and to M.Markina for typing the manuscript.

The work is supported by the Russian Fundamental Research Foundation, Grant No.93–02–14937 and by the INTAS, Grant No.94–2851.

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